Deriving Spacetime Geometry from Resolution Principles

Working Paper for Resolution Cosmology v5.3

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Abstract

This document develops the mathematical foundations for deriving the Schwarzschild spacetime metric from informational resolution principles.

We show that the full Schwarzschild metric—both weak-field and strong-field regimes—can be derived from the following premises: (1) proper time is accumulated resolution; (2) resolution has thermodynamic cost proportional to local temperature; (3) the Unruh effect provides the acceleration–temperature bridge at horizons where it is exact; (4) the Newtonian limit holds at large r; (5) the horizon location follows from pre-relativistic escape velocity physics.

A key advance of this version is that the resolution postulate itself implies the Tolman temperature relation as a consistency condition, eliminating the circularity present in earlier drafts where Tolman's result (originally derived within GR) had been imported. Version 5.3 further refines the strong-field derivation by restricting the Unruh effect to the horizon (where it is physically exact) and using the Michell dark star condition (1784) for horizon location—both pre-GR inputs.

With these gaps removed, the framework derives the Schwarzschild metric without invoking the Einstein field equations.

Extending this framework to cosmology, we reinterpret dark energy as the late-time acceleration of geometric resolution as thermal cost declines. The long-standing Coincidence Problem naturally dissolves. The model generates falsifiable predictions for the redshift evolution of the dark energy equation-of-state parameter w(z).

This is a working paper and remains open to comment.

Changelog

v5.3 (December 2025): Revised Section III to address the global Unruh inconsistency. The Unruh effect is now restricted to the horizon where it is physically exact. Added the Michell dark star condition (1784) for horizon location as an explicit pre-GR input. Clarified the uniqueness argument.

v5.2 (December 2025): Derived Tolman relation internally from resolution postulate, eliminating circularity.

I. Foundational Equations

A. Resolution Cost Function

From Landauer's principle, committing one bit of information to a thermal environment at temperature T requires minimum energy

$$C_{\text{res}} = k_B T \ln 2$$

where k_B is Boltzmann's constant. In the Resolution Cosmology framework, we adopt C_{res} as the thermodynamic cost of resolving one bit of quantum possibility into geometric record. Resolution is treated as thermodynamic erasure of unrealized alternatives, motivating the use of Landauer's bound.

B. Central Postulate: Proper Time as Accumulated Resolution

Fundamental Postulate: Proper time is accumulated resolution.

$$d\tau = \frac{dN}{\Gamma_0}$$

where dN is the number of bits resolved and Γ_0 is a reference resolution rate measured at infinity.

The local resolution rate depends on available free energy and local resolution cost:

$$\Gamma_{\text{local}} = \frac{\dot{E}_{\text{free}}}{k_B T_{\text{local}} \ln 2}$$

In a static spacetime with no net heat flow, $\dot{E}_{\text{free}}$ is constant for stationary worldlines. Thus,

$$\Gamma_{\text{local}} \propto \frac{1}{T_{\text{local}}}$$

C. Unruh Effect as the Acceleration–Temperature Bridge

From quantum field theory in flat spacetime, an observer with proper acceleration a experiences a thermal bath at

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

We treat this Unruh temperature as physically real for accelerating (non-inertial) detectors. **Important limitation**: The Unruh effect is exact only in flat spacetime or in local Rindler approximations. It is not valid globally in curved spacetime. In this framework, we apply it only at horizons where the geometry is precisely Rindler.

II. Weak-Field Derivation

A. Temperature Gradient

For $|GM/(rc^2)| \ll 1$, gravitational redshift follows from energy conservation:

$$\frac{E_2}{E_1} = 1 + \frac{\Phi_2 - \Phi_1}{c^2}, \quad \Phi(r) = -\frac{GM}{r}$$

For a thermal bath in equilibrium,

$$T(r) = T_\infty \left(1 + \frac{GM}{rc^2}\right)$$

B. Deriving g_{00}

Local resolution rate scales inversely with temperature:

$$\frac{\Gamma_{\text{local}}}{\Gamma_\infty} = \frac{T_\infty}{T(r)} \approx 1 - \frac{GM}{rc^2}$$

Proper time satisfies $d\tau = \sqrt{g_{00}}\, dt$. Using the postulate,

$$\sqrt{g_{00}} = \frac{\Gamma_{\text{local}}}{\Gamma_\infty} \approx 1 - \frac{GM}{rc^2}$$

Thus,

$$g_{00} \approx 1 - \frac{2GM}{rc^2}$$

the weak-field Schwarzschild limit derived without GR.

III. Strong-Field Derivation: Revised Approach

A. The Previous Gap

Earlier versions of this derivation assumed the Unruh relation $T_{local} = \hbar a/(2\pi k_B c)$ holds globally for stationary observers at all radii. However, the Unruh effect is strictly valid only in flat spacetime or in local Rindler approximations. Applying $T_{local} \propto a(r)$ globally leads to a differential equation yielding linear metrics (constant-acceleration Rindler geometry), not the 1/r Schwarzschild form.

We now present a revised derivation that restricts Unruh to where it is physically exact—the horizon—and uses additional pre-relativistic physics to close the gap.

B. Key Insight: Resolution Implies Tolman

This step remains unchanged and solid. From the metric definition and resolution postulate:

Metric definition: $$d\tau = \sqrt{g_{00}} \, dt$$

Resolution postulate: $$d\tau = \frac{dN}{\Gamma_0}$$

Bits resolved per coordinate time: $$dN = \Gamma_{\text{local}} \, dt = \Gamma_0 \frac{T_\infty}{T_{\text{local}}} dt$$

Combining these: $$d\tau = \frac{T_\infty}{T_{\text{local}}} dt$$

Comparing with the metric definition: $$\sqrt{g_{00}} = \frac{T_\infty}{T_{\text{local}}}$$

Rearranging yields the Tolman relation: $$T_{\text{local}} \sqrt{g_{00}} = T_\infty$$

This is derived from resolution principles, not imported from GR.

C. Horizon as Resolution Phase Boundary

In the resolution framework, proper time accumulates through the thermodynamic process of resolving quantum possibilities into geometric record. The resolution cost per bit is:

$$C_{\text{res}} = k_B T_{\text{local}} \ln 2$$

A horizon is defined as the surface where resolution cost diverges:

$$T_{\text{local}} \to \infty \implies C_{\text{res}} \to \infty \implies \Gamma_{\text{local}} \to 0$$

At the horizon, proper time cannot accumulate for stationary observers. This is not a coordinate artifact but a physical phase boundary: the edge of geometric recordability.

D. Horizon Location from Pre-Relativistic Physics

The horizon radius can be determined without invoking GR. In 1783-84, John Michell calculated the radius at which the Newtonian escape velocity equals the speed of light:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}} = c$$

Solving for r:

$$r_s = \frac{2GM}{c^2}$$

This "dark star" calculation predates general relativity by over a century. We adopt it as a boundary condition: the resolution phase boundary occurs at the Michell radius $r_s = 2GM/c^2$.

E. Regularity and Surface Gravity

For the geometry to be well-defined at the horizon (no conical singularity in the Euclidean continuation), the near-horizon metric must take the Rindler form. This regularity condition is not imported from GR but reflects geometric consistency: the resolution phase boundary must not introduce information discontinuities in the geometric record.

Near $r = r_s$, regularity requires:

$$g_{00}(r) \approx 2\kappa (r - r_s) + O((r-r_s)^2)$$

where κ is the surface gravity. From the Michell radius $r_s = 2GM/c^2$, geometric consistency gives:

$$\kappa = \frac{c^4}{4GM} = \frac{1}{2r_s}$$

(in geometric units where G = c = 1, this is $\kappa = 1/(4M) = 1/(2r_s)$).

F. Unruh at the Horizon

The Unruh effect is exact in Rindler spacetime. At the horizon, where the geometry is precisely Rindler, stationary observers (in the limiting sense) experience a thermal bath at:

$$T_H = \frac{\hbar \kappa}{2\pi k_B c} = \frac{\hbar c^3}{8\pi k_B GM}$$

This is the Hawking temperature, but derived here from: (1) the Michell radius, (2) regularity, and (3) Unruh in the exact Rindler regime—without invoking black hole thermodynamics or the Einstein equations.

Clarification: $T_H = \kappa/(2\pi)$ is the temperature measured at infinity (the asymptotic Hawking temperature). The local temperature near the horizon diverges as $T_{local} = T_H/\sqrt{g_{00}} \to \infty$, consistent with the resolution phase boundary interpretation.

G. Temperature Propagation via Tolman

The Tolman relation (derived in Section III.B) propagates the temperature throughout the spacetime:

$$T(r) = \frac{T_H}{\sqrt{g_{00}(r)}}$$

where $T_H = \kappa/(2\pi)$ is the temperature measured at infinity. Near the horizon, $T(r) \to \infty$ as expected. At large r, $T(r) \to T_H$ as $g_{00} \to 1$.

H. Deriving the Full Schwarzschild Metric

We now have sufficient constraints to uniquely determine $g_{00}(r)$:

1. **Near-horizon behavior** (from regularity): $g_{00} \approx 2\kappa(r - r_s) = (r - r_s)/r_s$

2. **Horizon location** (from Michell): $r_s = 2GM/c^2$

3. **Far-field behavior** (Newtonian boundary): $g_{00} \to 1 - 2GM/(rc^2)$ as $r \to \infty$

4. **Spatial metric** (from information density, Section IV): $g_{rr} = 1/g_{00}$

Uniqueness: Under spherical symmetry and vacuum (no informational sources, implying a harmonic resolution potential), the only solution matching both the near-horizon linear behavior and the far-field 1/r falloff is the monopole term:

$$g_{00}(r) = 1 - \frac{2GM}{rc^2} = 1 - \frac{r_s}{r}$$

Verification:

- At $r = r_s$: $g_{00} = 0$ ✓
- Near r_s: $g_{00} = (r - r_s)/r$, and $g_{00}'(r_s) = 1/r_s = 2\kappa$ ✓
- As $r \to \infty$: $g_{00} \to 1 - r_s/r$ (Newtonian) ✓

This is the Schwarzschild metric, derived without the Einstein field equations.

I. Summary of Inputs

Input	Source	GR-independent?
Proper time = accumulated resolution	Resolution postulate	Yes
Resolution cost = $k_B T \ln 2$	Landauer principle	Yes
Tolman relation	Derived from resolution	Yes
Horizon location $r_s = 2GM/c^2$	Michell (1784)	Yes (pre-GR)
Regularity (no conical singularity)	Geometric consistency	Yes
Unruh temperature at horizon	QFT in Rindler	Yes (flat spacetime QFT)
Newtonian far-field limit	Classical mechanics	Yes
$g_{rr} = 1/g_{00}$	Information density	Yes
Spherical symmetry & staticity	Assumed premises	Yes

No GR-specific inputs (Einstein equations, Ricci tensor, covariant conservation) are used.

IV. Spatial Curvature from Information Density

Thermal de Broglie wavelength:

$$\lambda_{\text{th}} \sim \frac{\hbar c}{k_B T}$$

We treat λ_{th} as the scale of distinguishable geometric information. Information density:

$$\rho_{\text{info}} \propto T$$

Traversing coordinate interval dr crosses more distinguishable states when $T_{local} > T_\infty$:

$$d\ell = \frac{T_{\text{local}}}{T_\infty} dr$$

Using Tolman:

$$d\ell = \frac{dr}{\sqrt{g_{00}}}$$

Thus,

$$g_{rr} = \frac{1}{g_{00}}$$

giving the full Schwarzschild spatial curvature.

V. Cosmological Extension

A. Resolution Cost Through Cosmic History

CMB temperature evolves as $T(a) = T_0/a$, giving resolution cost

$$C(a) = \frac{k_B T_0 \ln 2}{a}$$

Early universe: high cost; late universe: low cost.

B. Dark Energy as Accelerated Resolution

As cost drops, geometric production accelerates:

$$\dot{N} = \frac{\dot{E}_{\text{free}}}{k_B T \ln 2}$$

This late-time acceleration is not a repulsive force but increased geometric efficiency as the universe cools.

C. De Sitter as Constant-Rate Resolution

Constant expansion rate H = constant requires constant $\dot{N}$. A cosmological constant acts as constant free-energy density, providing this condition as T declines.

D. Coincidence Problem Dissolved

Matter domination and dark-energy domination become natural sequential thermodynamic phases. We observe the transition precisely because the universe has cooled past the acceleration threshold.

E. Predictions for w(z)

If $\dot{E}_{free}$ is not constant:

- $\dot{E}_{free}$ drops faster than T: $w > -1$
- $\dot{E}_{free}$ drops slower than T: $w < -1$

Falsifiable prediction: Next-generation surveys (DESI, Euclid, Roman) should detect evolution in $w(z)$ correlated with cosmic thermodynamic state.

VI. Black Hole Implications

A. Horizon as Resolution Phase Boundary

As $r \to r_s$:

$$a(r) \to \infty, \quad T \to \infty, \quad C \to \infty, \quad \Gamma \to 0$$

Resolution freezes. Proper time cannot accumulate for stationary states.

B. Interior Structure

The resolution cost is maximal everywhere inside. Time (as accumulated resolution) cannot progress, preventing classical singularity formation. Selective resolution, experimentally observed in Lüders-type quantum measurement, provides a microphysical precedent.

C. Information and Hawking Radiation

Hawking temperature emerges naturally from the framework (Section III.F). As T_H rises during evaporation, resolution cost falls, enabling information release. Black holes store information at maximal density until evaporation.

VII. Open Problems

1. **Non-spherical solutions**: Kerr and Reissner–Nordström require additional information channels (angular momentum, charge) with their own thermodynamic costs.

2. **Einstein equations**: This framework derives the Schwarzschild metric, not the full field equations. Can $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ emerge from global consistency of the geometric record?

3. **Free energy source**: Late-universe $\dot{E}_{free}$ may arise from horizon degrees of freedom, vacuum fluctuations, or gravitational potential energy.

VIII. Conclusion

We have derived the complete Schwarzschild metric from resolution principles alone. The derivation chain is:

1. Proper time = accumulated resolution
2. Resolution cost = $k_B T \ln 2$
3. Resolution postulate ⇒ Tolman relation
4. Horizon location from Michell (1784): $r_s = 2GM/c^2$
5. Regularity ⇒ near-horizon Rindler form
6. Unruh at horizon (where exact) ⇒ $T_H = \kappa/(2\pi)$
7. Tolman propagates temperature; Newtonian boundary at infinity
8. Uniqueness under vacuum spherical symmetry ⇒ $g_{00} = 1 - r_s/r$
9. Information density ⇒ $g_{rr} = 1/g_{00}$

The key advances are: (a) deriving Tolman's relation internally, eliminating dependence on GR results; (b) restricting Unruh to the horizon where it is exact; (c) using the pre-relativistic Michell condition for horizon location. Extensions to cosmology reinterpret dark energy as late-time resolution efficiency and dissolve the Coincidence Problem.

Appendix A: Detailed Derivation Notes

1. Recovering g_{00} Without GR

The derivation uses:

- metric definition of proper time
- Landauer cost
- Unruh temperature (at horizon only)
- equilibrium thermodynamics
- Newtonian boundary conditions
- Michell escape velocity condition

No GR-specific inputs (Einstein equations, curvature tensors, covariant conservation) are used.

2. Why Tolman Must Hold

The Tolman relation arises because:

$$d\tau = dN/\Gamma_0, \quad dN = \Gamma_{\text{local}} dt, \quad \Gamma_{\text{local}} \propto 1/T$$

Thus,

$$d\tau = \frac{T_\infty}{T_{\text{local}}} dt$$

Consistency with $d\tau = \sqrt{g_{00}}\, dt$ forces Tolman.

3. Why Schwarzschild is Unique

Static, spherically symmetric, vacuum metrics have two degrees of freedom. The resolution framework provides:

- Tolman's constraint (from resolution postulate)
- Information density constraint ($g_{rr} = 1/g_{00}$)
- Near-horizon behavior (from regularity)
- Far-field behavior (from Newtonian limit)

These overdetermine the system, leaving only the Schwarzschild solution.

Appendix B: Relation to Thermodynamic Gravity

Jacobson (1995) derived Einstein's equations from local Clausius relations applied to Rindler horizons. Padmanabhan and Verlinde developed related thermodynamic pictures.

This work differs fundamentally:

1. Jacobson assumes GR horizon thermodynamics; we do not.
2. Our derivation uses Unruh temperature (at horizons) + Landauer cost, not GR area–entropy proportionality.
3. We derive a specific metric (Schwarzschild), not the Einstein tensor.
4. Tolman relation is obtained from informational consistency, not from GR.
5. Horizon location comes from Michell (1784), not from GR.

Thus this framework is parallel to, not derivative of, thermodynamic gravity approaches.

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